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Conical Spring and Localised Region Methodologies for Modelling of Soft Tissue Deformation

A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

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Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; any editorial work, paid or unpaid, carried out by a third party is acknowledged; and, ethics procedures and guidelines have been followed.

Mohd Nadzeri Omar

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Abstract

Considerable research efforts have been dedicated to the development of virtual reality simulators that facilitate medical students in learning anatomy and surgery in the virtual environment and to allow surgeons to rehearse the surgical procedures. The level of realism depends upon the simulation accuracy and the computational efficiency of underlying deformable models. Ideally, the deformable models should be able to simulate accurately mechanical behaviours of soft tissues with real-time visual and force feedback.

Modelling soft tissue deformation is not an easy task. Due to the complexity of soft tissue properties, many methods have been proposed to model soft tissue properties. One of the most well-known methods is the Finite Element Method (FEM). In this method, the soft tissue is represented by multiple elements that are derived based on complex mathematical formulations. It has been proven that the method is able to simulate soft tissue properties accurately, but it requires high computational cost to produce real-time interaction. In this regard, the Mass Spring Method (MSM) has been proposed as an alternative. The traditional MSM model simulates soft tissue deformation by discretising the soft tissue into several mass points that are connected to each other by linear springs. The major advantage of MSM is it has an excellent computational performance. However, the MSM application is limited to linear deformation, which does not represent the actual behaviour of the soft tissue deformation.

In this thesis, an improved MSM model has been proposed to simulate the complex behaviour of soft tissue deformations. The improved MSM model is called conical spring model which considers the general behaviour of soft tissue deformation that is a combination of linear and nonlinear responses. Piecewise approach is used to discretise each response individually, and the conical spring methodology is used to model the deformation behaviours during all the responses. The piecewise approach gives precision in modelling while the conical spring methodology that was founded on stiffness variation, has improved the accuracy of the simulation due to its ability to model any type of linear and nonlinear responses. Moreover, the generated conical spring model is based on the force propagation approach. The computational performance of the model relies on the number of nodes involved in the propagation of the force. Inherently, computational time can be improved by considering the nodes only in a deformation area, and ignoring the other nodes.

Soft tissue deformation commonly occurs only within a local region. As the effect of the deformation outside the local region is very little, it can be ignored in real practice. In this thesis, methods to define the local region were proposed. The methods are based on the linear elastic theory. As reported in Chapter 4 of this thesis, the localised region was generated based on displacement value induced when the simulation model was subjected to an external load. The Boussinesq method, which is widely used in the soil mechanics, was used to estimate the induced displacement value. However, the Boussinesq method is limited to the isotropic material. Therefore, as described in Chapter 5, the study has extended the isotropic localised region to anisotropic localised region by introducing an anisotropic factor which was derived based on cross-anisotropic properties. By using the anisotropic factor, the anisotropic localised region is determined from the corresponding isotropic case.

Alternatively, in Chapter 6, we have presented a localised region that was generated based on stress value induced during a loading process. It is shown for point load type of contact, in comparison to ABAQUS analysis, stress based localisation has a better accuracy than the displacement based localisation. However, the stress value that is also determined using the Boussinesq method, has no relation to the material properties. Hence, a combination of the Hertzian and the Boussinesq method was used to generate localised regions with respect to the material properties and loading conditions.

In the final chapter, contributions of the study were discussed, and some of the future works to expand the research were listed out.

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Table of Contents

ABSTRACT ____ III
ACKNOWLEDGEMENT ____ V
TABLE OF CONTENTS ____ VI
LIST OF TABLES ____ VIII
LIST OF FIGURES ____ IX

1 INTRODUCTION ____ 12

1.1 BACKGROUND ____ 14
1.2 OBJECTIVES ____ 17
1.3 CONTRIBUTION OF THE DISSERTATION ____ 17
1.4 DISSERTATION ORGANISATION ____ 18

2 LITERATURE REVIEW ____ 20

2.1 SOFT TISSUE PROPERTIES ____ 20
2.2 MODELLING METHODS ____ 22
2.2.1 Discrete mechanics ____ 23
2.2.2 Continuum mechanics ____ 24
2.2.3 Comparison of MSM and FEM ____ 25
2.3 IMPROVEMENT IN MSM MODEL ____ 26
2.3.1 Nonlinear behaviour ____ 27
2.3.2 Optimisation process ____ 31
2.3.3 Computational Time ____ 32
2.3.4 Conclusion ____ 33
2.4 CHAPTER SUMMARY ____ 34

3 SOFT TISSUE MODELLING USING CONICAL SPRING METHODOLOGY ____ 35

3.1 CONICAL SPRING METHODOLOGY ____ 35
3.1.1 Fundamental of cylindrical spring stiffness ____ 35
3.1.2 Parameters of conical spring ____ 39
3.1.3 Types of conical spring ____ 39
3.1.4 Formulation of conical spring ____ 41
3.1.5 Stiffness variation in conical spring ____ 42
3.1.6 Conical spring deflection behaviours ____ 44
3.1.7 Model development ____ 46
3.1.8 Piecewise approach ____ 49
3.2 ANALYSIS AND DISCUSSION ____ 51
3.2.1 Mechanical behaviour ____ 51
3.2.2 Comparison to experimental data ____ 58
3.2.3 Comparison to traditional MSM ____ 61
3.2.4 Interactive feedback ____ 62
3.3 CHAPTER SUMMARY ____ 63

4 LOCALISED REGION USING ESTIMATION OF DISPLACEMENT FOR ISOTROPIC MATERIAL WITH KNOWN INPUT LOAD ____ 64

4.1 ANALYSIS ON DISPLACEMENT DISTRIBUTION ____ 64

4.2 METHOD 68
4.3 LOCALISED REGION 70
4.4 ANALYSIS 72
4.4.1 Behaviour analysis 72
4.4.2 Performance analysis 75
4.5 DISCUSSION 79
4.6 CHAPTER SUMMARY 79

5 LOCALISED REGION USING ESTIMATION OF DISPLACEMENT FOR ANISOTROPIC MATERIAL WITH KNOWN INPUT LOAD 81

5.1 ANALYSIS ON DISPLACEMENT DISTRIBUTION 82
5.2 METHOD 85
5.3 LOCALISED REGION 90
5.4 ANALYSIS 91
5.4.1 Behaviour analysis 91
5.4.2 Performance analysis 93
5.5 DISCUSSION 95
5.6 CHAPTER SUMMARY 96

6 LOCALISED REGION USING ESTIMATION OF STRESS WITH KNOWN INPUT DISPLACEMENT 97

6.1 BEHAVIOUR ANALYSIS 97
6.1.1 Different material 101
6.1.2 Different load 102
6.2 METHOD 102
6.3 LOCALISED REGION 106
6.4 ANALYSIS 107
6.4.1 Comparison between stress and displacement estimation 107
6.4.2 Validation of Hertzian method 108
6.4.3 Behaviour analysis 109
6.4.4 Computational performance 112
6.5 DISCUSSION 114
6.6 CHAPTER SUMMARY 115

7 CONCLUSIONS AND FUTURE WORK 116

LIST OF PUBLICATIONS 120

REFERENCES 121

APPENDICES 125

APPENDIX 1 125
APPENDIX 2 130
APPENDIX 3 132

List of Tables

Table 3.1. Different sets of conical spring parameters used to generate the three phases of soft tissue deformation as presented in Fig. 3.12. ____ *50*
Table 3.2. The relative error of the proposed method in comparison with reference data. ____ *54*
Table 3.3. Details of the conical spring parameters used for the red and white lattices. ____ *57*
Table 3.4. Material specifications of the Silicon Ecoflex 00-30. ____ *58*
Table 3.5. Update time of the conical spring model and traditional MSM. ____ *62*
Table 4.1. Parameters of the localised region at different input load. ____ *73*
Table 4.2. The parameters used and the outcomes of the comparison between the proposed method and FEM. _ *76*
Table 4.3. Computational performance of conical spring model with different sizes of localised region. ____ *77*
Table 5.1. Material properties of cross-anisotropic finite element model. ____ *82*
Table 5.2. Definition of parameters for the localised region in Configuration 1. ____ *92*
Table 5.3. The parameters used and the outcomes of the comparison between the proposed method and FEM. _ *94*
Table 6.1. Validation of the Hertzian method with the FEM model. ____ *109*
Table 6.2. Computational time for stress and displacement based localisation with 396 nodes. ____ *114*

List of Figures

Figure 1.1. Minimally Invasive Surgery (MIS). 12
Figure 1.2. Robot-Assisted Minimally Invasive Surgery [Photo © The da Vinci® Surgical System]. 13
Figure 1.3. Components of surgical simulations [Photo © The da Vinci® Surgical System]. 14
Figure 1.4. Soft tissue properties. 15
Figure 1.5. Soft tissue modelling. 16
Figure 2.1. Stress-strain curve for soft tissue [16]. 20
Figure 2.2. Graph showing the independence lamellar-level elastic modulus of trabecular tissue from age and gender [17]. 21
Figure 2.3. Values of Poisson ratio, shear modulus, and bulk modulus vs. trans-pulmonary pressure at 20, 40, and 60 years [17]. 22
Figure 2.4. An example of a structure for MSM models. 23
Figure 2.5. A one-dimensional chainmail structure undergoing deformation [23]. 24
Figure 2.6. An example of a structure of a FEM model and types of element. 24
Figure 2.7. The nonlinear deformation behaviour is taken from Mohammadi [52]. 28
Figure 2.8. The nonlinear MSM using spring configuration given by Gaizka et al. [36]. 29
Figure 2.9. The nonlinear deformation behaviour, as taken from Choi et al. [58]. 29
Figure 2.10. The nonlinear deformation behaviour, taken from Basafa et al. [31]. 30
Figure 2.11. The localisation of the deformable object [37]. 33
Figure 3.1. The structure of the cylindrical spring [104]. 36
Figure 3.2. The cross-sectional element of spring wire under torsion [104]. 36
Figure 3.3. Parameterisation of conical spring. 40
Figure 3.4. Projection view of the conical spring structure. 43
Figure 3.5. The projection view of conical spring at different ratio of the smallest and the largest coil $(D1/D2)$*. a) Large ratio, b) Medium ratio, and c) Small ratio.* 43
Figure 3.6. Deformation behaviour of conical spring at different number of coils involved within a given displacement value. 44
Figure 3.7. Deformation behaviour of conical spring at different values of diameter ratio $D1/D2$*.* 45
Figure 3.8. Deformation behaviour of conical spring at different values of shear modulus G. 46
Figure 3.9. Conical spring model structure. 47
Figure 3.10. Connection illustration for the hexahedral element. 47
Figure 3.11. Dynamic process of conical spring model. 49
Figure 3.12. Load-displacement relationship of conical spring model using the piecewise approach. 51
Figure 3.13. Image of the conical spring model during deformation. 52
Figure 3.14. Deformation behaviour of conical spring model in comparison with the reference data of Type 1 [71]. 52
Figure 3.15. Deformation behaviour of conical spring model in comparison with the reference data of Type 1 [71]. 53
Figure 3.16. Deformation behaviour of conical spring model in comparison with the linear function given in Eq. (3.36). 53
Figure 3.17. Deformation of a kidney model develops using conical spring model. 54
Figure 3.18. The viscoelastic properties of the conical spring model. a) Hysteresis, b) Creep, and c) Force relaxation. 56
Figure 3.19. Anisotropic deformation of the conical spring model. 58
Figure 3.20. The compression test using the commercial INSTRON Compression machine. 59
Figure 3.21. Images of the compression test on the soft tissue phantom (Ecoflex 00-30). 59
Figure 3.22. The outcome of the compression test on the soft tissue phantom for six specimens. 60

Figure 3.23. Deformation behaviour of the conical spring model in comparison to the experimental data obtained from the compression test on the soft tissue phantom of Specimen 5. 61
Figure 3.24. Deformation behaviour of traditional MSM and the conical spring model. 62
Figure 3.25. Implementation of the proposed method in SOFA open source. Image of the original liver model followed by deformed model. 63
Figure 4.1. Total displacement distribution inside isotropic body subjected to normal loading. 65
Figure 4.2. Total, horizontal and vertical displacement components of the isotropic body subject to normal loading. 65
Figure 4.3. Total displacement distribution of FEM model at different input load measured vertically underneath the contact point. 66
Figure 4.4. Total displacement distribution of FEM model at different elastic modulus measured vertically underneath the contact point. 66
Figure 4.5. The total displacement of the model using different Poisson ratio value measured along vertical axis underneath the contact point. 67
Figure 4.6. The total displacement of the model at different Poisson ratio value measured along the surface parallel to the contact point. 67
Figure 4.7. Illustration of the localised region. 71
Figure 4.8. The process of generating localised region. 72
Figure 4.9. Vertical displacement along the vertical axis underneath the contact point. 73
Figure 4.10. The relationship between the elastic modulus and distance of the small displacement $u\ min$. 74
Figure 4.11. Displacement distribution of the model using different Poisson ratio values measured along: a) The vertical axis underneath the contact point (vertical), and b) The surface parallel to the contact point (horizontal). 75
Figure 4.12. The displacement distribution recorded by the proposed method and the FEM simulation. 75
Figure 4.13. Deformation of conical spring model with different sizes of localised region, a) Model 1 (small) and b) Model 2 (large). 77
Figure 4.14. The computational performance of the proposed localised region with regards to the size of the localised region interpreted by the number of nodes. 78
Figure 4.15. Deformation of conical spring model with different input loads, a) Load = 10 N, and b) Load = 20 N. 78
Figure 5.1. The illustration of isotropy and different types of anisotropy structure. a) Isotropic structure, b) Orthotropic structure, and c) Cross-anisotropic structure. 81
Figure 5.2. Total displacement distribution for a different set of material properties. a) Isotropic, b) Cross-anisotropic Set 1, and c) Cross-anisotropic Set 2. 83
Figure 5.3. Total displacement distribution for isotropic, and material of Set 1 and Set 2 measured a) Along the vertical direction, and b) At the surface. 83
Figure 5.4. Total, horizontal and vertical displacement components of isotropic and cross-anisotropic body subject to normal loading. 84
Figure 5.5. Comparison of total displacement and vertical displacement component measured a) At the surface, and b) Along vertical direction. 85
Figure 5.6. The process of generating localised region for cross-anisotropic materials. 91
Figure 5.7. The relationship between the anisotropic dimensionless factor and anisotropic intensity. 92
Figure 5.8. Relationship of the dimensionless factor and the distance where $umin$ occurs for the parameter lz. 93
Figure 5.9. The deformation behaviour of an isotropic conical spring model with localised region a) Isotropic localised region and b) Cross-anisotropic localised region with anisotropic factor equal to 1.5. The colour contour explains the magnitude of displacement occurs. 94
Figure 6.1. Image of the ABAQUS set up and the corresponding deformation of the FEM model. 98
Figure 6.2. Image of the Von Mises stress distribution. 99
Figure 6.3. Images of stress distribution for each stress component. a) S11, horizontal stress in 11 direction, b) S22, vertical stress in 22 direction, c) S33, horizontal stress in 33 direction, and d) S12, shear stress in 12 direction. 100

Figure 6.4. The vertical stress distributions plotted along the vertical axis underneath the contact point with different elastic modulus values. ______ *101*
Figure 6.5. The result of the FEM simulation using indentation model for the vertical stress distribution plotted along the vertical axis underneath the contact point under different Poisson ratio. ______ *101*
Figure 6.6. The vertical stress distribution plotted along the vertical axis underneath the contact point with different loading values. ______ *102*
Figure 6.7.The superposition of point load for circular pressure. ______ *104*
Figure 6.8. Contact interaction between the indenter and material under an external load F, where Q is the contact pressure value at some displacement d. ______ *104*
Figure 6.9. The localised region for stress based approach in the form of hemisphere volume. ______ *106*
Figure 6.10. The process of generating the stress based localised region with a known input displacement. ______ *107*
Figure 6.11. The vertical stress distribution plotted along the vertical axis underneath the contact point. ______ *108*
Figure 6.12. The vertical displacement distribution plotted along the vertical axis underneath the contact point. *108*
Figure 6.13. The vertical stress distribution with different contact pressures (input displacements). ______ *110*
Figure 6.14. The vertical stress distribution with different elastic modulus values. ______ *110*
Figure 6.15. The vertical stress distribution with different Poisson ratios. ______ *111*
Figure 6.16. The images of the conical spring model during compression. a) Conical spring model without localised region, and b) Conical spring model with localised region. ______ *111*
Figure 6.17. The vertical displacement distribution underneath the centre point of the circular contact area with a comparison to the FEM model. ______ *113*
Figure 6.18. The vertical stress distribution underneath the centre point of the circular contact area with a comparison to the FEM model. ______ *113*
Figure 6.19. Image of conical spring model under deformation using a) Stress based, and b) Displacement based localisation approach. ______ *114*
Figure A2.0.1. Plane stress element. ______ *130*

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